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CALCULATION OF NONLINEAR FUNCTIONS FROM CERTAIN VARIABLES IN STUDYING THE STABILITY OF AN AUTOMATIC REGULATION SYSTEM

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/Figures are appended./

This article analyzes those conditions necessary for the convergence of automatic regulation processes, following given but not "small" initial deflections. The method proposed earlier by the author is extended to the case where the differential equations of the process contain any number of nonlinear functions, some of them in several variables.

Generalizing somewhat the terminology introduced in the author's preceding work, "Convergence of Processes of Automatic Regulation after Large Initial Deflections," Avtomatika i Telemekhanika, 2 - 3, 1946, we shall consider hereafter that the process of regulation undergoes a decrement in region L, if the equilibrium stabilized by the regulator and upset as a result of an initial disturbance, characterized by any point in region L, is re-established some time later after cessation of the disturbing effect. The regulation process undergoes a decrement in L, if the equilibrium position stabilized by the regulator is asymptotically stable and if all points belonging to the given region L of initial deflections also belong, for the most part, to the region of stability of this equilibrium position.

Conditions sufficient for a decrement of the system were determined by the author in his above-mentioned work for the case where the equations describing the regulation process contain one nonlinear function in one variable, i.e., reducing to the form:

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$$\dot{x}_1 = \sum_{j=1}^{j=n} a_{1j} x_j + f(x_k),$$

$$\dot{x}_1 = \sum_{j=1}^{j=n} a_{ij} x_j,$$

where a_{1j} and a_{ij} are constants, (some of which are zero), k is any number 1, 2, ..., n, and i is any number 2, 3, ..., n.

The generalization of the criteria found in author's above-mentioned work for the case where the system of equations describing the process contains several nonlinear functions, each in one variable, is not difficult.

The generalization of the same criteria for the case where the equations of the process contain any number of nonlinear functions, including also functions in several variables, is the object of the present report. (Since the present report is a continuation of his above-mentioned work, the author has refrained from duplicating here a survey of the preceding works pertaining to the same problem of automatic regulation.)

Generalization of Criterion I

Let us assume that the process of regulation is described by any system of equations:

$$\dot{x}_i = F_i(x_1, x_2, \ldots, x_n), \qquad i = 1, 2, \ldots, n \tag{1}$$

where F_i are assigned functions of all or some of the specified variables; x_1, x_2, ..., x_n are increments of the generalized coordinates and velocities, reckoned from their equilibrium values in a regulated equilibrium position in such a way that the origin of the coordinates of the system's phase space (1) ($x_1 = x_2 = \ldots = 0$) corresponds to this equilibrium and therefore $F_i(0, \ldots, 0) = 0$.

Let us simultaneously examine with equations (1) the linear equations

$$\dot{x}_i = \sum_{j=1}^{j=n} a_{ij} x_j, \qquad i = 1, 2, \ldots, n \tag{2}$$

where a_{ij} are constants (some of which are zero).

Concerning the system of equations (2) we shall assume only that it satisfies Routh-Hurwitz' criteria. Otherwise, the coefficients a_{ij} may be selected arbitrarily and this arbitrariness may be utilized in a practical application of the method given below. (Thus the relationships (2) need not be equations of "small fluctuations" relative to system (1).)

The considerations which permit us to generalize our criterion I are correct generalizations of the considerations brought forth during the proving of this criterion, as shown in the preceding work of the author.

Let us arbitrarily take the quadratic form

$$U_A = \sum_{j=1}^{j=n} \sum_{k=1}^{k=n} A_{x_i x_k} x_i x_k, \tag{3}$$

making the constant coefficients $A_{x_i x_k}$ such that form (3) will be definitely negative.

Let us take a second quadratic form

$$V = \sum_{k=1}^{k=n} \sum_{i=1}^{i=n} B_{x_i x_k} x_i x_k, \tag{4}$$

and determine its coefficients $B_{x_i x_k}$ from the equations which we will obtain by equating the coefficients having similar terms in the relationship

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$$U_A=\frac{dV}{dt} \quad \text{or} \quad \sum_{i=1}^{i=n}\sum_{k=1}^{k=n} A_{x_i x_k} x_i x_k=\sum_{i=1}^{i=n}\frac{\partial V}{\partial x_i}\dot{x}_i, \qquad (5)$$

where $\dot{x}_i$ is taken in accordance with equations (2).

The phase curves of system (2) intersect any of the ellipsoids of the series $V=R$ (R is any nonnegative number) only from the outside inward, since on the strength of (5) we have $\frac{dV}{dt}=U_A$ and $U_A=$ everywhere negative.

Let us examine now the system of equations

$$\dot{x}_i=\sum_{j=1}^{j=n}(a_{ij}+\breve{a}_{ij})x_j, \qquad i=1,2,\dots,n \qquad (6)$$

where a_{ij} has the same values as in (2) and $\breve{a}_{ij}$ are numbers determined below.

Retaining in (4) the above-found values of coefficients Bx_ix_k, we shall find the derivative

$$\frac{dV}{dt}=\sum_{i=1}^{i=n}\frac{dV}{dx_i}\dot{x}_i, \qquad (7)$$

taking the values $\dot{x}_i$ from system (6).

Then equation (7) will establish the derivative $\frac{dV}{dt}$ as a quadratic form relative to variables $x_1, x_2, \dots, x_n$. Coefficients of this form (let us designate them Sx_ix_n) will depend on $\breve{a}_{ij}$, and the inequalities which have to be satisfied in order that the quadratic form (7) be definitely negative will bring conditions which in this case must be satisfied by $\breve{a}_{ij}$:

$$\breve{a}_{i}^{*}<\breve{a}_{ij}<\breve{a}_{ij}^{**}, \qquad i,j=1,2,\dots,n \qquad (8)$$

where the quantities $\breve{a}_{ij}^{*}$ and $\breve{a}_{ij}^{**}$ are well defined (It is essential only that $\breve{a}_{ij}=0$ should satisfy inequalities (8).) by the coefficients a_{ij} and Ax_ix_k, which may be selected arbitrarily, limited only by the above-indicated conditions.

System (6), together with inequalities (8), defines the set of linear equations whose phase curves intersect any of the ellipsoids of the system $V=R$ only from the outside inward.

Let us now substitute in the derivative

$$\frac{dV}{dt}=\sum_{i=1}^{i=n}\frac{\partial V}{\partial x_i}\dot{x}_i$$

values $\dot{x}_i$ determined by the examined system (1). It is not difficult to notice that $\frac{dV}{dt}$ continues to remain negative at any point in the phase space, if the quantities $\breve{a}_{ij}$ satisfying the inequality can be so selected (8) that at this point the value of any of the nonlinear functions $F_i(x_1, x_2, \dots, x_n)$ coincide with the values of the corresponding linear function

$$\sum_{j=1}^{j=n}(a_{ij}+\breve{a}_{ij})x_j.$$

We shall select from among the ellipsoids of the series $V=R$ any ellipsoid $V=R_1$, and name the series of points inside this region as "region R."

If all $F_i(x_1,x_2, \dots, x_n)$ of the investigated nonlinear system are such that, for any point $x_{1o}, x_{2o}, \dots, x_{no}$ of region R_1 of the phase space, it is possible to find, among the quantities $\breve{a}_{ij}$ satisfying inequalities (8), such values (perhaps for each of its points) that

$$F_i(x_{1_0}, x_{2_0}, \dots, x_{n_0}) = \sum_{j=1}^{j=n} (a_{ij} + \tilde{a}_{ij}) x_j, \quad i = 1, 2, \dots, n \tag{9}$$

then the derivative $\frac{dV}{dt}$ throughout the region R_1 expressed by system (1) is definitely negative and the curves of system (1) intersect the set of ellipsoids $V = R$ only from outside inward, converging toward the origin of the coordinates and, consequently, system (1) in this case undergoes a decrement in region R_1.

Generalizing the considerations somewhat, we can put the result obtained in the form of the following generalized criterion I:

If it is possible for a linear system of differential equations with constant coefficients (2) to construct a definitely positive Lyapunov [Liapounoff] function V (the term "Lyapunov function" is here understood to be in the restricted meaning indicated in the author's above-mentioned work, and not in the broader sense in which it is usually understood in this connection) whose derivative $\frac{dV}{dt}$ expressed by system (2) will be a definitely negative function for any values of a_{ij} satisfying the inequalities

$$a_{ij}^{*} < a_{ij}^{**}, \qquad i, j = 1, 2, \dots, n$$

then, for the nonlinear system (1) to undergo a decrement in any region $V = R_1$ of the phase space, it will be sufficient. This is true, if, for every point of this region, it is possible to select such values of a_{ij} satisfying the indicated inequalities, at which the value of $\sum_{j=1}^{j=n} a_{ij} x_j$ coincides with the value of $F_i(x_1, x_2, \dots, x_n)$.

Numerical Example

1. Formulation of Example

Taking into account the nonlinearity of the characteristics of the engine and disregarding the nonlinearity of the characteristics of the regulator, the object of this example is to determine those conditions, for the installation described below consisting of a high-speed Diesel engine with direct speed regulation, which are sufficient to cause convergence of the regulation process after any initial deviation produced by a momentary change in the engine speed of not more than 100 rpm and a displacement of the regulator coupling of not more than 3 mm. (Similar initial deflections can be caused by short-duration load changes.) The initial speed of the coupling is equal to zero.

2. Description of Installation and Equation

The high-speed Diesel engine is utilized for fixed operation at $n = 1{,}200$ rpm. A resistance-load torque-moment is applied to the engine which in effect does not vary during momentary changes in engine speed and is equal to 16 kg·m of torque.

Figures 1 and 2 show two variants of engine characteristics. Each curve determines the variation in torque-moment ΔM_g (computed from equilibrium $M_g = 16$ kg·m) in the rpm range -300 to +300 rpm (computed from equilibrium $n_0 = 1{,}200$ rpm) and with an unchanging (but different) position of the coupling of the regulator, which is disconnected from the engine.

The moment of inertia I_g of all the engine's rotating and forward moving masses, brought out in its flywheel, equals 0.24 kg·m^2.

The equation of motion of the engine has the form

$$\dot{x} = \frac{30}{\pi I_g} \Delta M_g$$

or

$$\dot{x} = 39.9\, F(x, y),$$

where $F(x,y)$ is a function defined by the family of curves shown in Figure 3, first variant of the engine, and in Figure 4, second variant of the engine, obtained from Figures 1 and 2 by changing the scale of the ordinate axis 39.9 times.

A centrifugal regulator of construction NATI (Figure 5) is used in the capacity of a direct-action regulator. The parameters of the regulator are such (all data on NATI regulator were submitted to the author by Prof G. G. Kalish) that the equation of the regulator has the form

$$\ddot{y} + 100\dot{y} + 6872\, y - 104.6\, x = 0$$

In these equations and also in Figures 1, 2, 3, 4: $x = \Delta n$ are the deviations (increment-decrement) in the engine's rotation computed from equilibrium center 1,200 rpm ($x > 0$ if the rotations increase); y is the displacement of the regulator coupling, computed from the equilibrium position corresponding to engine operation at 1,200 rpm.

Thus, the process of regulation is described by the equations

$$\left.\begin{aligned} \dot{x} &= 39.9\, F(x, y), \\ \dot{y} &= \xi \\ \dot{\xi} &= -100\,\xi - 6872\, y + 104.6\, x \end{aligned}\right\} \qquad (10)$$

In the phase space with coordinates x, y, ξ, the region of initial deviations is given by the rectangle $-100 < x < +100$, $-3 < y < +3$ lying in the plane $\xi = 0$.

Determination of Conditions for a Decrement of the System

We shall take as a linear approximation of equations (10)

$$\left.\begin{aligned} \dot{x} &= ax - by, \\ \dot{y} &= \xi, \\ \dot{\xi} &= -100\,\xi - 6872\, y + 104.6\, x \end{aligned}\right\} \qquad (11)$$

placing $a = 0$ and $b = 50$ and taking into consideration the course of the curves in Figures 3 and 4.

This system is similar to system (21) of author's above-mentioned work, for which the set of ellipsoids of interest to us was constructed to intersect the curves of the linear system only from the outside inward.

In our case, $Na = 0$, $b = 50$, $h = 100$, $c = 6{,}872$ and $k = 104.6$. Repeating the computations carried out in that work, but taking into account these values for the coefficients and setting U_A in the form $U_A = -A(x^2 + y^2 + \xi^2)$ for the equation of the set of ellipsoids

$$V = \frac{1}{2}(B_x x^2 + B_y y^2 + B_\xi \xi^2) - (B_{xy} xy + B_{x\xi} x\xi + B_{y\xi} y\xi) = R, \qquad (12)$$

we easily determine the following values for the coefficients:

$$\left.\begin{aligned} B_x &= 1.355A, & B_y &= 70.74A, & B_\xi &= 0.010148A \\ B_{xy} &= 2.018A, & B_{x\xi} &= 0.0096A, & B_{y\xi} &= -0.0148A \end{aligned}\right\} \qquad (13)$$

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Let us now examine equations

$$\left.\begin{aligned} \dot{x} &= -N\tilde{a}x-(50-\tilde{b})y, \\ \dot{y} &= \xi \\ \dot{\xi} &= -100\xi-6872y+104.6x \end{aligned}\right\} \qquad (14)$$

and determine the derivative

$$\frac{dV}{dt}=\frac{\partial V}{\partial x}\dot{x}+\frac{\partial V}{\partial y}\dot{y}+\frac{\partial V}{\partial \xi}\dot{\xi},$$

using the values $\dot{x}$, $\dot{y}$, and $\dot{\xi}$ from system (14).

For this derivative, the quadratic form $U_s =$

$$U_s=-(S_x x^2+S_y y^2+S_\xi \xi^2+2S_{xy}xy+2S_{x\xi}x\xi+2S_{y\xi}y\xi), \qquad (15)$$

is employed with coefficients equal to

$$\left.\begin{aligned} &S_x=A(1+1.355\,N\tilde{a}),\ S_y=A(1-2.018\,\tilde{b}),\ S_\xi=A, \\ &S_{xy}=\frac{A}{2}(1.355\,\tilde{b}-2.018\,N\tilde{a}),\ S_{x\xi}=-\frac{A}{2}\,0.0096\,N\tilde{a}, \\ &S_{y\xi}=-\frac{A}{2}\,0.0096\,\tilde{b}. \end{aligned}\right\} \qquad (16)$$

The quadratic form (15) is definitely negative if

$$S_\xi>0,\ S_x S_\xi-S_{x\xi}^2>0$$

~~and~~

$$S_x S_y S_\xi+2S_{xy}S_{x\xi}S_{y\xi}-S_x S_{y\xi}^2-S_y S_{x\xi}^2-S_\xi S_{xy}^2>0.$$

On the strength of (16), these inequalities are satisfied if

$$(1+1.355\,N\tilde{a})-0.0096^2 N\tilde{a}^2>0$$

and

$$4.072\,N\tilde{a}^2+(5.47\tilde{b}-5.42)N\tilde{a}+(1.336\tilde{b}^2+8.072\tilde{b}-4)<0.$$

It is easily demonstrated that any $N\tilde{a}$ and $\tilde{b}$ satisfy these inequalities, if

$$\left.\begin{aligned} -0.2 &\leqslant N\tilde{a} \leqslant 0.3 \\ -4 &\leqslant \tilde{b} \leqslant 0.36 \end{aligned}\right\} \qquad (17)$$

To determine whether the conditions for the above-proved criteria are fulfilled, we must now compare, passing over to equations (10), the nonlinear function $F = 39.9F(x,y)$ given by the series of curves in Figure 3 (or in Figure 4) with the linear function

$$F=-N\tilde{a}x-(50+\tilde{b})y$$

It is necessary to establish for which values of x and y it is possible to select $N\tilde{a}$ and $\tilde{b}$ satisfying the inequalities (17) in such a way that

$$39.9\,F(x,y)=-N\tilde{a}x-(50+\tilde{b})y.$$

With this purpose in mind, a family of straight lines $F=a_1^*x-50.36y$ and $F=a_2^*x-46y$ were drawn, (in Figure 6 for first variant of the engine and in Figure 7 for the second variant) where

$$a_1^*=0.3,\ \text{if}\ xy\leqslant 0\ \text{and}\ a_1^*=-0.2\ \text{if}\ xy>0,$$
$$a_2^*=0.3,\ \text{if}\ xy>0\ \text{and}\ a_2^*=-0.2\ \text{if}\ xy\leqslant 0,$$

and y was given all those values, in turn, for which the curves illustrated in Figures 2 and 3 were drawn (these values are $y = -3, -2, -1, 0, 1, 2, 3$).

Thus, each of the indicated values for y corresponds, on the one hand, to a specific curve in Figure 2 (or in Figure 3) and, on the other hand, to a region limited by four segments of straight lines (for case $y = 0$ with two straight lines), shown in Figure 6 (or in Figure 7).

One of these regions (corresponding to $y = -3$) is cross-hatched in Figure 6 and 7. (the number of these regions to be constructed is the same as the number of curves assigned to the nonlinear function F (x,y) being considered)

Conditions for our criterion are fulfilled in this case if all the curves assigned to function F (x,y) do not emerge beyond the limits of the regions constructed for them in this manner.

Solution of Example

From Figure 6, it follows that in regulating an engine whose characteristics are shown in Figure 2, the conditions for our criterion are fulfilled and the process of regulation converges after any initial deflection whatever they might be (within limits, determined by the range of fixed characteristics).

In regulating an engine whose characteristics are shown in Figure 3, the conditions of our criterion, in accordance with Figure 7, are fulfilled only for $x > -200$.

Let us separate from among the ellipsoids (12), with values B determined by equations (13) that ellipsoid related to plane $x = -200 = \text{const}$, and let us cut this ellipsoid by the plane $\xi = 0$.

In the cross section there appears an ellipse

$$0.6775x^2 + 35.37y^2 - 2.018xy = 27795.8.$$

The region of initial deflections, assigned by the conditions of the example

$$\left.\begin{array}{l} -100 < x < +100 \\ -3 < y < +3 \end{array}\right\} \qquad (18)$$

lies entirely within this ellipse. Accordingly, the tested regulation process described by equations (10) converges after any initial deflection satisfying the given conditions (18).

Stating a New Problem

The region through which the nonlinear characteristics can pass arbitrarily without violating the conditions necessary for a decrement of the system becomes more extensive, the stronger the inequalities in the values of a_{ij}^* and a_{ij}^{**} entering (8). It is natural, therefore, to explain the limiting values for these quantities. At the same time, it is no longer necessary to connect them with the proved criterion and to require that system (6) use the same Lyapunov function for any a_{ij} satisfying the equalities (8). It is essential only that system (1) should undergo a decrement in the entire space, if a_{ij} can be selected for any point in such a way that the values in the right side of equations (1) and (6) coincide.

Let system (2) satisfy Routh-Hurwitz' criterion for any a_{ij}, if only

$$b^{*}_{ij} < a_{ij} < b^{**}_{ij}, \qquad i,j = 1, 2, \ldots, n$$

and does not satisfy it, if $a_{ij} < b^{*}_{ij} - \varepsilon$ or $a_{ij} > b^{**}_{ij} + \varepsilon$ however small the positive number ε may be.

Is it at all possible to widen the band between a^{*}_{ij} and a^{**}_{ij} up to the limits of Routh-Hurwitz; i.e., up to the values b^{*}_{ij} and b^{**}_{ij} ?

We shall limit ourselves here merely to the statement of this interesting problem, whose solution would not only simplify considerably the above-stated method, but also would develop fundamentally the basis of linear methods, which the author considers the most important result of the present work.

[Appended figures follow:]

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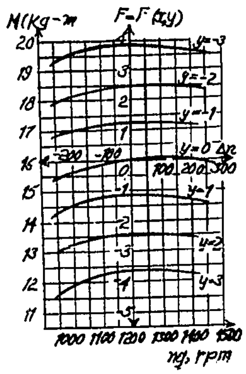

Figure 1
Given Characteristics of a Diesel Engine (first variant)

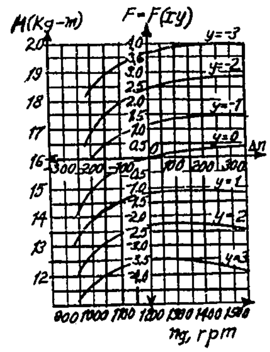

Figure 2
Given Characteristics of a Diesel Engine (second variant)

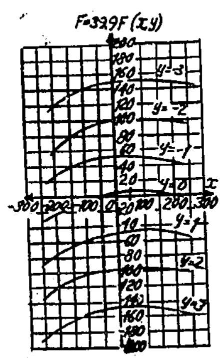

Figure 3
Value of the Function 39.9 F (x,y) for a Diesel Engine (first variant)

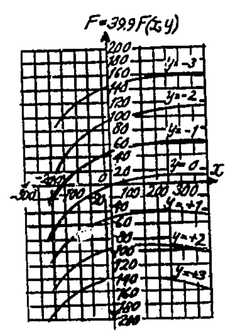

Figure 4
Value of the Function 39.9 F (x,y) for a Diesel Engine (second variant)

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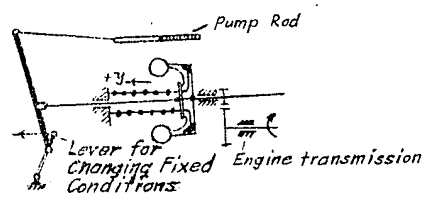

Figure 5
Principle Diagram of Diesel Engine Regulator

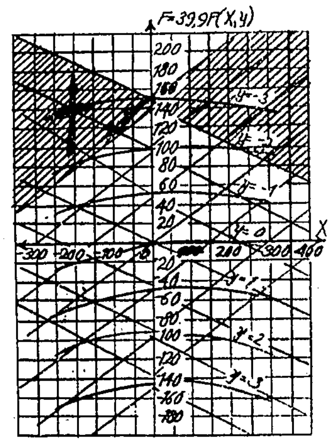

Figure 6
Comparison of Diesel Engine Characteristics (first variant) With Regions Through Which These Characteristics Must Pass for the Regulation Process to Converge After Any Initial Deflection

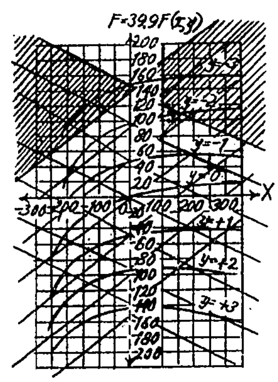

Figure 7
Comparison of Diesel Engine Characteristics (second variant) With Regions Through Which These Characteristics Must Pass for the Regulation Process to Converge After Any Initial Deflection

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